

# Statistical Arbitrage for Multiple Co-Integrated Stocks

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## Appendix A Supplementary Materials

This is the Supplementary Material<sup>1</sup> for our article entitled to Statistical Arbitrage for Multiple Co-Integrated Stocks<sup>2</sup>.

Figure A.1 and Figure A.2 exhibit the optimal wealth of unconstrained portfolio and market-neutral constrained portfolio for varying sliding window lengths over the entire time interval from January 3<sup>rd</sup>, 2000 to May 6<sup>th</sup>, 2021.

Table A.1 to Table A.4 illustrate the annualised statistics of the myopic wealth and the optimal wealth for the unconstrained portfolio among four different sub-intervals over the entire time interval from January 3<sup>rd</sup>, 2000 to May 6<sup>th</sup>, 2021.

Table A.5 to Table A.8 display the annualised statistics of the myopic wealth and the optimal wealth for the market-neutral constrained portfolio among four different sub-intervals over the entire time interval from January 3<sup>rd</sup>, 2000 to May 6<sup>th</sup>, 2021.

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<sup>1</sup><https://apapani.wordpress.ncsu.edu/publications>

<sup>2</sup><https://arxiv.org/abs/1908.02164>

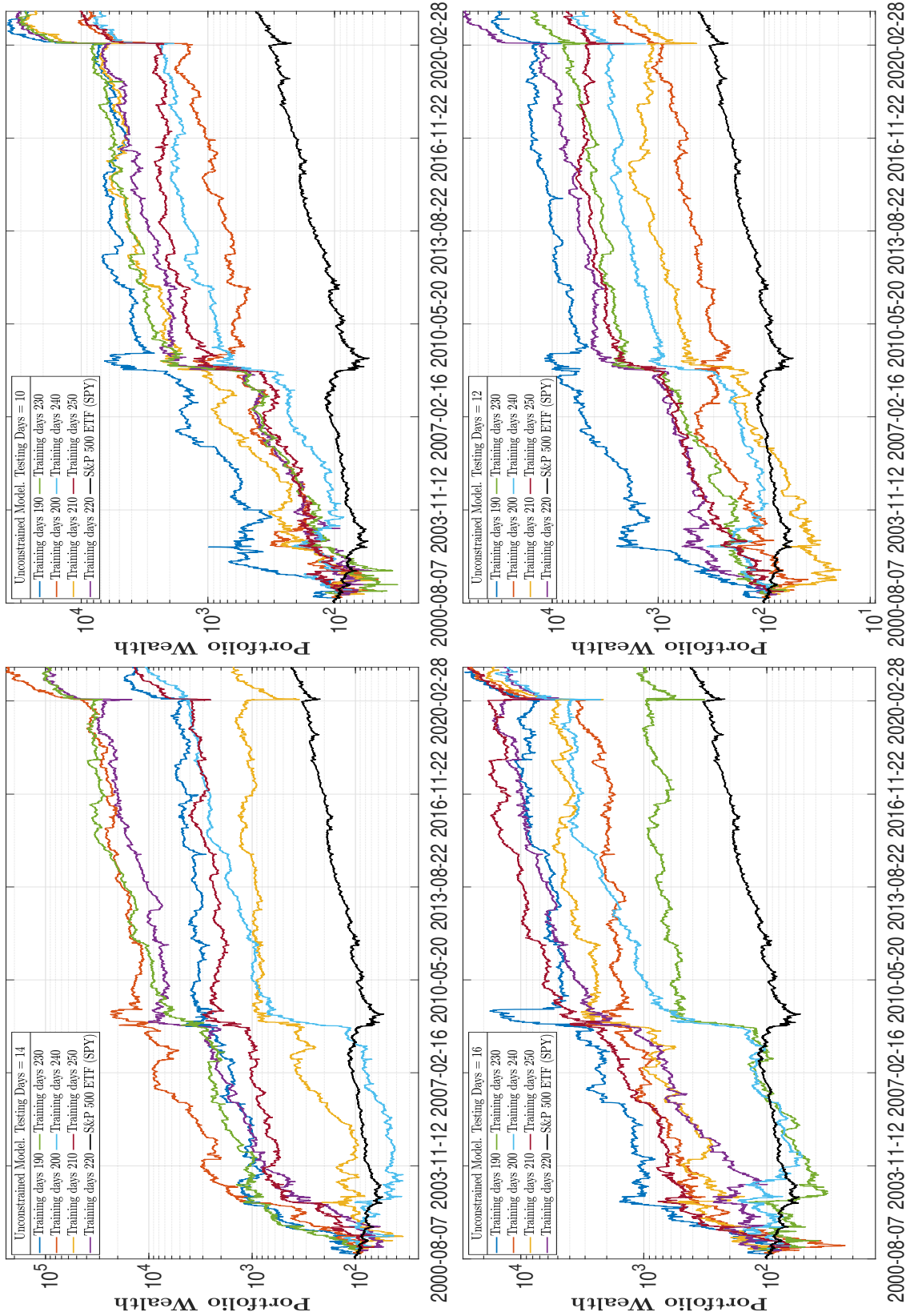


Figure A.1: Optimal wealth trajectories of out-of-sample tests with different taring and testing windows for the unconstrained portfolios during testing period from January 3<sup>rd</sup>, 2000 to May 6<sup>th</sup>, 2021. The vertical axis is in logarithmic scale with base ten. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ .

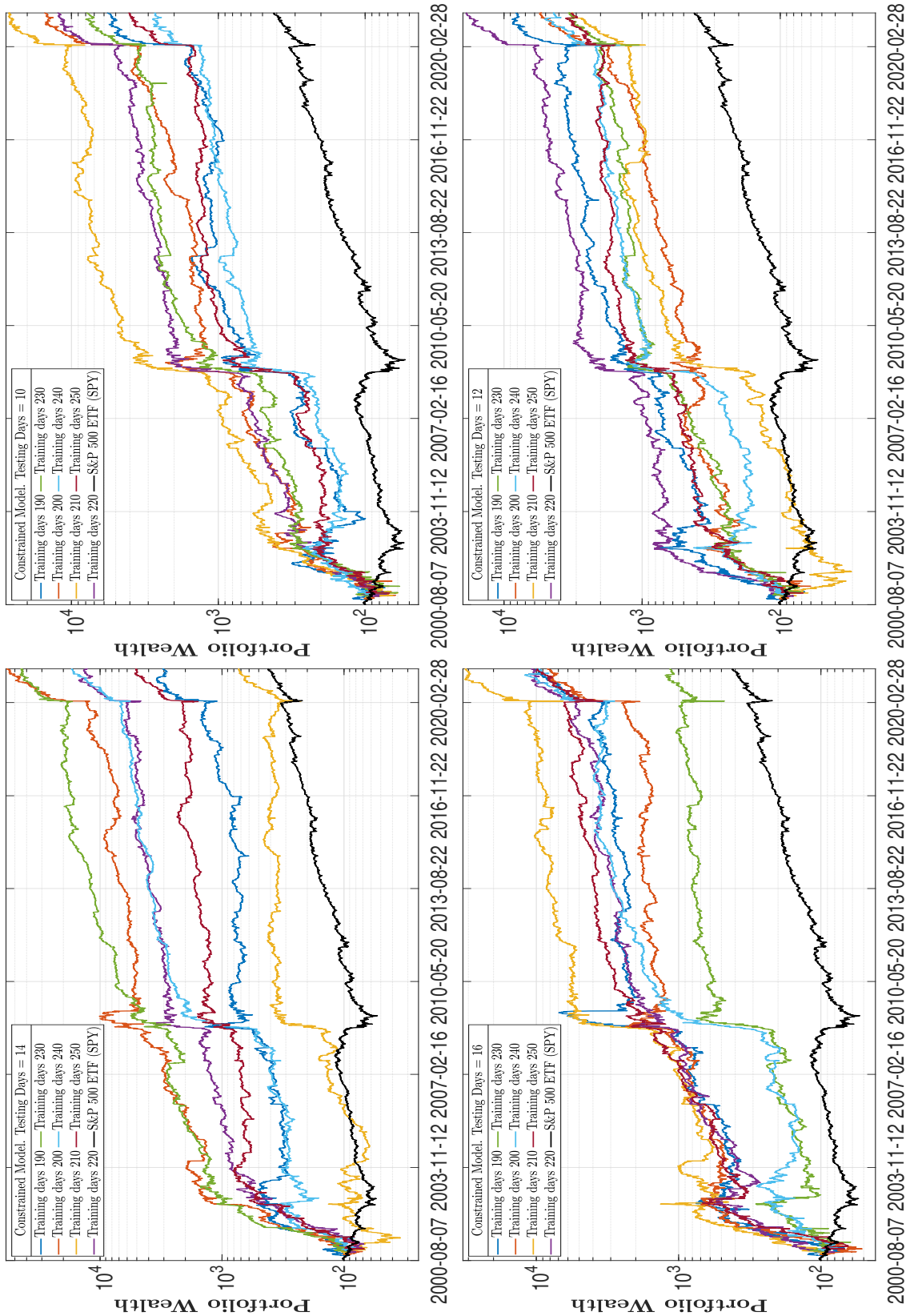


Figure A.2: Optimal wealth trajectories of out-of-sample tests with different taring and testing windows for the market-neutral constrained portfolios during testing period from January 3<sup>rd</sup>, 2000 to May 6<sup>th</sup>, 2021. The vertical axis is in logarithmic scale with base ten. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ .

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{100 E \left( \frac{\Delta W_t}{W_t} \right)}{\Delta t}$		$\frac{0.01(\text{Return} - r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	269.189	762.973	0.202	0.599	27.514	60.595	1.311	0.994	0.236	0.683
200	10	175.525	239.890	0.224	0.652	22.285	45.156	0.950	0.677	0.252	0.604
210	10	151.609	377.430	0.198	0.532	20.026	45.037	0.959	0.828	0.226	0.586
220	10	161.404	217.664	0.201	0.599	20.855	41.913	0.986	0.683	0.213	0.509
230	10	126.343	222.963	0.199	0.593	18.022	41.636	0.855	0.685	0.231	0.731
240	10	105.592	64.080	0.181	0.494	15.820	22.042	0.819	0.426	0.230	0.610
250	10	156.828	189.011	0.179	0.460	20.217	32.144	1.076	0.677	0.243	0.606
190	12	434.238	3042.101	0.232	0.616	35.367	86.106	1.481	1.381	0.268	0.573
200	12	285.006	215.004	0.203	0.625	28.381	42.271	1.351	0.660	0.212	0.644
210	12	65.927	-12.171	0.219	0.598	12.299	15.673	0.515	0.245	0.244	0.820
220	12	198.079	476.881	0.218	0.521	23.721	48.340	1.042	0.909	0.208	0.443
230	12	187.898	300.986	0.241	0.573	23.615	43.701	0.938	0.745	0.206	0.531
240	12	114.100	43.943	0.173	0.475	16.471	18.614	0.894	0.371	0.205	0.735
250	12	204.540	544.735	0.172	0.467	23.447	48.015	1.307	1.007	0.178	0.435
190	14	203.056	1184.984	0.245	0.649	24.675	71.424	0.967	1.084	0.346	0.552
200	14	450.696	4240.781	0.244	0.634	36.294	93.593	1.446	1.461	0.302	0.422
210	14	253.394	92.338	0.239	0.607	27.449	31.511	1.106	0.503	0.269	0.673
220	14	260.782	1365.562	0.200	0.503	27.155	65.187	1.307	1.277	0.233	0.465
230	14	430.667	1917.827	0.195	0.511	34.681	72.071	1.731	1.391	0.171	0.368
240	14	94.496	-38.989	0.207	0.523	15.209	4.384	0.685	0.065	0.217	0.732
250	14	274.299	855.966	0.200	0.527	28.009	58.516	1.353	1.091	0.239	0.405
190	16	168.095	1400.592	0.253	0.677	22.445	76.018	0.846	1.107	0.276	0.567
200	16	175.055	825.837	0.245	0.635	22.742	63.564	0.887	0.985	0.402	0.779
210	16	361.257	426.935	0.242	0.653	32.849	54.512	1.316	0.820	0.320	0.783
220	16	267.150	467.307	0.229	0.649	28.128	55.330	1.182	0.837	0.282	0.638
230	16	210.867	-9.694	0.230	0.690	24.924	22.927	1.042	0.318	0.269	0.772
240	16	63.550	-11.986	0.245	0.577	12.623	14.244	0.475	0.229	0.338	0.703
250	16	425.794	1088.499	0.200	0.509	34.719	61.585	1.684	1.191	0.165	0.474

Table A.1: Annualised statistics of out-of-sample tests for sub-interval [2000-01-03, 2005-10-13] of myopic wealth and optimal wealth for the unconstrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 5.078%, volatility of 0.186, expected return of 2.693%, Sharpe ratio of 0.091, and maximum drawdown of 0.421.

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{0.01(\text{Return}-r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	328.705	373.889	0.211	0.430	30.559	39.425	1.400	0.894	0.196	0.595
200	10	121.789	70.556	0.218	0.413	17.902	18.758	0.774	0.430	0.240	0.608
210	10	141.629	384.428	0.225	0.387	19.781	38.253	0.836	0.962	0.303	0.370
220	10	246.856	555.269	0.216	0.392	26.671	44.349	1.189	1.106	0.186	0.338
230	10	362.727	946.501	0.192	0.347	31.923	52.030	1.614	1.472	0.250	0.324
240	10	287.123	478.952	0.201	0.373	28.649	41.468	1.373	1.086	0.161	0.322
250	10	226.053	470.042	0.187	0.340	25.026	40.038	1.287	1.147	0.190	0.371
190	12	205.653	115.748	0.241	0.474	24.623	26.048	0.978	0.529	0.264	0.612
200	12	61.391	8.194	0.242	0.429	12.243	10.782	0.464	0.228	0.282	0.546
210	12	241.410	685.224	0.232	0.428	26.634	48.988	1.106	1.122	0.237	0.361
220	12	241.752	625.207	0.224	0.386	26.510	45.952	1.140	1.165	0.168	0.298
230	12	353.907	624.522	0.186	0.352	31.415	44.989	1.636	1.248	0.108	0.308
240	12	358.201	823.563	0.186	0.330	31.641	48.959	1.645	1.455	0.155	0.291
250	12	217.307	427.378	0.206	0.355	24.851	38.956	1.159	1.070	0.226	0.369
190	14	208.058	155.534	0.403	0.675	28.880	37.660	0.693	0.543	0.345	0.504
200	14	195.598	192.751	0.242	0.466	24.077	31.892	0.954	0.663	0.350	0.582
210	14	232.307	335.514	0.243	0.430	26.355	37.557	1.042	0.850	0.327	0.436
220	14	274.081	451.882	0.225	0.386	28.314	40.697	1.215	1.028	0.180	0.371
230	14	166.951	397.124	0.229	0.423	21.891	40.281	0.914	0.929	0.259	0.397
240	14	463.927	1513.044	0.190	0.333	35.805	60.064	1.831	1.773	0.172	0.195
250	14	115.122	125.401	0.219	0.374	17.422	22.873	0.751	0.584	0.177	0.379
190	16	412.601	216.950	0.365	0.662	37.871	42.750	1.011	0.630	0.277	0.771
200	16	138.883	97.803	0.251	0.458	20.097	23.752	0.760	0.496	0.237	0.591
210	16	240.030	402.973	0.292	0.470	28.095	42.166	0.928	0.875	0.341	0.462
220	16	253.144	722.470	0.234	0.390	27.461	48.807	1.129	1.225	0.267	0.339
230	16	214.321	505.117	0.208	0.391	24.634	42.820	1.136	1.069	0.327	0.381
240	16	484.534	1273.849	0.215	0.377	37.044	58.652	1.675	1.528	0.195	0.358
250	16	216.752	486.400	0.218	0.381	25.075	41.991	1.106	1.076	0.261	0.425

Table A.2: Annualised statistics of out-of-sample tests for sub-interval [2005-10-14, 2010-12-17] of myopic wealth and optimal wealth for the unconstrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 17.725%, volatility of 0.247, expected return of 6.232%, Sharpe ratio of 0.212, and maximum drawdown of 0.552.

Statistics Windows		Profit (%) $100 \left( \frac{W_T - W_0}{W_0} \right)$	Volatility $\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$	Expected Return (%) $100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$	Sharpe Ratio $\frac{0.01(\text{Return} - r)}{\text{Volatility}}$	Maximum Drawdown $\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$
Train	Test	Myopic	Myopic	Myopic	Myopic	Myopic
		Optimal	Optimal	Optimal	Optimal	Optimal
190	10	14.969	0.113	3.364	0.209	0.242
200	10	30.881	0.114	5.908	0.429	0.173
210	10	83.136	0.118	12.539	0.975	0.168
220	10	64.793	0.111	10.407	0.846	0.160
230	10	59.327	0.109	9.748	0.800	0.170
240	10	71.362	0.103	11.131	0.981	0.152
250	10	32.537	0.091	5.970	0.545	0.126
190	12	83.201	0.125	12.579	0.927	0.182
200	12	80.920	0.117	12.257	0.963	0.097
210	12	83.273	0.115	12.515	1.003	0.212
220	12	62.768	0.106	10.108	0.860	0.171
230	12	71.306	0.108	11.154	0.944	0.183
240	12	86.698	0.108	12.868	1.096	0.161
250	12	46.957	0.086	7.962	0.809	0.095
190	14	28.173	0.144	5.879	0.338	0.205
200	14	51.696	0.121	8.872	0.649	0.199
210	14	52.760	0.126	9.088	0.640	0.162
220	14	127.425	0.124	16.867	1.285	0.188
230	14	152.067	0.107	18.739	1.654	0.132
240	14	117.853	0.104	15.863	1.427	0.111
250	14	64.242	0.101	10.298	0.917	0.123
190	16	60.928	0.132	10.138	0.695	0.195
200	16	28.547	0.140	5.886	0.349	0.263
210	16	30.035	0.137	6.080	0.370	0.249
220	16	111.749	0.127	15.504	1.145	0.120
230	16	46.677	0.121	8.256	0.600	0.150
240	16	138.608	0.109	17.705	1.534	0.120
250	16	98.097	0.098	13.959	1.326	0.118

Table A.3: Annualised statistics of out-of-sample tests for sub-interval [2010-12-20, 2016-02-25] of myopic wealth and optimal wealth for the unconstrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 75.089%, volatility of 0.156, expected return of 12.066%, Sharpe ratio of 0.711, and maximum drawdown of 0.186.

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{0.01(\text{Return}-r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	212.181	426.093	0.225	0.333	24.517	37.519	1.045	1.095	0.233	0.317
200	10	339.630	901.051	0.242	0.351	31.567	50.656	1.263	1.414	0.203	0.280
210	10	233.906	331.616	0.263	0.376	26.641	35.009	0.975	0.904	0.264	0.375
220	10	280.673	748.468	0.216	0.320	28.341	46.668	1.268	1.429	0.135	0.202
230	10	221.258	390.567	0.267	0.361	26.206	37.282	0.942	1.005	0.288	0.354
240	10	172.723	298.623	0.218	0.318	21.933	31.884	0.961	0.973	0.162	0.230
250	10	163.417	341.385	0.193	0.269	20.793	32.623	1.027	1.177	0.141	0.259
190	12	50.150	140.883	0.388	0.503	14.086	27.401	0.337	0.525	0.473	0.535
200	12	161.066	318.941	0.277	0.391	22.017	34.466	0.758	0.855	0.327	0.470
210	12	18.386	41.694	0.238	0.361	6.337	13.828	0.224	0.356	0.602	0.726
220	12	326.129	815.175	0.215	0.312	30.553	47.938	1.372	1.502	0.165	0.222
230	12	141.034	248.719	0.354	0.473	23.007	34.789	0.622	0.714	0.541	0.650
240	12	57.033	96.510	0.228	0.334	11.441	18.870	0.458	0.535	0.444	0.584
250	12	35.746	33.755	0.272	0.379	9.271	11.964	0.304	0.290	0.467	0.617
190	14	108.210	206.520	0.411	0.541	21.159	33.870	0.490	0.607	0.514	0.581
200	14	410.313	1038.297	0.291	0.426	35.483	55.307	1.183	1.275	0.273	0.352
210	14	3.799	20.057	0.250	0.365	4.093	10.793	0.124	0.269	0.641	0.762
220	14	211.682	368.207	0.320	0.435	26.752	38.448	0.804	0.861	0.432	0.552
230	14	128.878	203.056	0.224	0.318	18.620	26.568	0.788	0.804	0.225	0.306
240	14	147.784	317.318	0.281	0.382	21.181	34.181	0.718	0.869	0.338	0.392
250	14	157.352	265.662	0.206	0.286	20.604	29.470	0.954	0.996	0.266	0.386
190	16	108.200	193.025	0.378	0.546	20.262	32.932	0.510	0.585	0.476	0.580
200	16	463.357	1106.716	0.277	0.386	37.297	55.606	1.311	1.415	0.189	0.233
210	16	181.377	354.925	0.496	0.623	29.935	44.782	0.584	0.703	0.553	0.634
220	16	91.467	42.404	0.299	0.475	16.982	18.267	0.535	0.364	0.538	0.709
230	16	10.268	25.775	0.366	0.485	8.172	15.457	0.196	0.298	0.600	0.688
240	16	140.840	267.735	0.296	0.420	21.255	33.522	0.685	0.775	0.420	0.525
250	16	29.985	66.953	0.215	0.326	7.602	15.702	0.307	0.450	0.501	0.622

Table A.4: Annualised statistics of out-of-sample tests for sub-interval [2016-02-26, 2021-05-06] of myopic wealth and optimal wealth for the unconstrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 135.484%, volatility of 0.185, expected return of 18.257%, Sharpe ratio of 0.932, and maximum drawdown of 0.337.

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{100 E \left( \frac{\Delta W_t}{W_t} \right)}{\Delta t}$		$\frac{0.01(\text{Return} - r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	147.205	92.481	0.152	0.472	18.799	24.060	1.174	0.489	0.261	0.685
200	10	236.604	437.831	0.171	0.514	25.146	46.024	1.415	0.876	0.170	0.465
210	10	170.163	657.454	0.137	0.386	20.384	47.097	1.417	1.195	0.161	0.401
220	10	183.112	419.189	0.141	0.415	21.388	41.089	1.447	0.966	0.103	0.384
230	10	145.382	350.275	0.141	0.442	18.635	39.548	1.248	0.871	0.140	0.495
240	10	97.543	79.146	0.129	0.383	14.221	18.740	1.028	0.463	0.155	0.507
250	10	130.837	165.659	0.123	0.346	17.252	25.254	1.321	0.701	0.082	0.256
190	12	135.862	416.451	0.178	0.467	18.319	42.871	0.972	0.897	0.277	0.575
200	12	282.812	319.205	0.167	0.512	27.606	41.145	1.593	0.785	0.171	0.487
210	12	92.010	3.476	0.161	0.457	14.061	11.261	0.810	0.225	0.177	0.720
220	12	202.504	679.238	0.148	0.359	22.786	46.688	1.470	1.272	0.131	0.283
230	12	151.525	288.733	0.202	0.474	20.097	37.816	0.948	0.776	0.174	0.392
240	12	117.597	84.197	0.129	0.363	16.132	18.643	1.169	0.486	0.214	0.650
250	12	141.794	413.747	0.130	0.345	18.254	38.258	1.330	1.081	0.141	0.360
190	14	90.001	212.098	0.194	0.497	14.410	34.790	0.692	0.680	0.274	0.504
200	14	350.254	2035.505	0.201	0.517	31.393	73.036	1.514	1.393	0.268	0.356
210	14	120.863	2.746	0.175	0.470	17.005	11.534	0.913	0.224	0.255	0.672
220	14	249.528	979.734	0.157	0.394	25.747	54.276	1.579	1.352	0.129	0.302
230	14	374.764	2090.374	0.159	0.415	31.863	69.140	1.940	1.643	0.088	0.250
240	14	183.131	236.359	0.167	0.394	21.862	31.605	1.247	0.776	0.169	0.525
250	14	195.180	652.399	0.149	0.409	22.454	48.109	1.443	1.152	0.195	0.323
190	16	107.599	457.913	0.215	0.542	16.551	48.192	0.723	0.871	0.337	0.518
200	16	190.655	638.836	0.187	0.488	22.584	51.002	1.155	1.024	0.208	0.494
210	16	305.670	716.582	0.188	0.503	29.188	54.052	1.497	1.054	0.321	0.697
220	16	249.729	648.011	0.189	0.460	26.289	49.805	1.340	1.061	0.295	0.528
230	16	157.616	90.372	0.181	0.547	20.229	27.898	1.062	0.492	0.211	0.650
240	16	80.214	68.784	0.168	0.422	12.976	19.168	0.715	0.431	0.290	0.502
250	16	260.325	703.142	0.166	0.416	26.666	49.793	1.542	1.172	0.203	0.527

Table A.5: Annualised statistics of out-of-sample tests for sub-interval [2000-01-03, 2005-10-13] of myopic wealth and optimal wealth for the market-neutral constrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 5.078%, volatility of 0.186, expected return of 2.693%, Sharpe ratio of 0.091, and maximum drawdown of 0.421.



Statistics Windows		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{0.01(\text{Return}-r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
190	10	271.079	391.537	0.182	0.347	27.155	36.831	1.433	1.033	0.154	0.417
200	10	174.447	191.263	0.177	0.336	21.214	26.255	1.143	0.753	0.150	0.481
210	10	233.614	579.354	0.167	0.308	24.942	42.130	1.436	1.336	0.158	0.263
220	10	180.011	368.381	0.164	0.302	21.509	34.689	1.248	1.117	0.175	0.291
230	10	162.804	391.854	0.141	0.260	19.974	34.642	1.347	1.296	0.214	0.280
240	10	193.044	307.372	0.144	0.285	22.172	31.540	1.465	1.073	0.089	0.285
250	10	158.641	350.665	0.141	0.268	19.719	33.233	1.328	1.201	0.174	0.337
190	12	172.776	249.840	0.178	0.326	21.102	29.625	1.132	0.878	0.178	0.293
200	12	54.100	43.971	0.170	0.301	9.902	11.784	0.523	0.358	0.249	0.403
210	12	256.209	615.959	0.168	0.321	26.237	43.442	1.501	1.324	0.172	0.259
220	12	137.000	279.802	0.165	0.297	18.243	30.436	1.046	0.992	0.154	0.235
230	12	172.518	213.405	0.121	0.257	20.427	25.758	1.607	0.962	0.090	0.366
240	12	262.011	531.115	0.146	0.266	26.358	39.670	1.732	1.454	0.118	0.340
250	12	98.504	215.748	0.127	0.242	14.326	25.582	1.049	1.017	0.141	0.230
190	14	114.012	125.642	0.280	0.463	18.300	25.151	0.617	0.521	0.328	0.592
200	14	156.631	155.613	0.188	0.381	20.123	25.552	1.019	0.644	0.214	0.546
210	14	190.472	243.485	0.180	0.338	22.452	29.643	1.189	0.847	0.203	0.303
220	14	134.003	202.423	0.189	0.320	18.388	26.629	0.918	0.800	0.164	0.338
230	14	124.876	269.697	0.167	0.315	17.305	30.525	0.978	0.939	0.245	0.391
240	14	271.538	784.733	0.163	0.292	27.114	46.956	1.600	1.576	0.154	0.222
250	14	70.851	95.526	0.143	0.269	11.567	16.824	0.741	0.588	0.149	0.288
190	16	353.246	327.648	0.264	0.490	32.633	39.707	1.200	0.790	0.201	0.729
200	16	143.586	124.098	0.200	0.367	19.345	22.452	0.916	0.584	0.180	0.495
210	16	311.656	609.489	0.187	0.343	29.403	43.993	1.516	1.255	0.140	0.288
220	16	101.771	237.584	0.187	0.323	15.503	29.019	0.775	0.866	0.228	0.384
230	16	136.717	270.919	0.169	0.320	18.334	30.741	1.027	0.930	0.283	0.379
240	16	376.044	1035.434	0.162	0.301	31.979	52.187	1.916	1.699	0.166	0.240
250	16	156.792	344.392	0.160	0.291	19.871	33.590	1.179	1.119	0.298	0.428

Table A.6: Annualised statistics of out-of-sample tests for sub-interval [2005-10-14, 2010-12-17] of myopic wealth and optimal wealth for the market-neutral constrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 17.725%, volatility of 0.247, expected return of 6.232%, Sharpe ratio of 0.212, and maximum drawdown of 0.552.

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{0.01(\text{Return} - r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	6.066	13.841	0.085	0.120	1.510	3.253	0.060	0.188	0.253	0.337
200	10	24.633	40.880	0.083	0.117	4.644	7.377	0.438	0.544	0.143	0.180
210	10	46.065	65.069	0.088	0.127	7.805	10.609	0.769	0.759	0.128	0.174
220	10	34.125	44.128	0.078	0.109	6.055	7.764	0.652	0.618	0.097	0.124
230	10	30.275	38.710	0.080	0.113	5.516	7.071	0.564	0.535	0.153	0.189
240	10	39.881	54.280	0.077	0.108	6.901	9.120	0.766	0.749	0.196	0.265
250	10	16.668	20.130	0.064	0.091	3.247	4.029	0.349	0.333	0.101	0.140
190	12	54.444	70.014	0.093	0.132	8.901	11.212	0.852	0.773	0.182	0.260
200	12	65.483	83.743	0.083	0.114	10.175	12.529	1.110	1.010	0.074	0.102
210	12	29.749	38.434	0.087	0.120	5.474	7.083	0.515	0.508	0.213	0.282
220	12	55.893	64.379	0.075	0.108	8.985	10.326	1.061	0.861	0.101	0.152
230	12	36.468	32.553	0.082	0.113	6.443	6.181	0.665	0.457	0.140	0.195
240	12	50.922	67.277	0.081	0.112	8.428	10.749	0.913	0.870	0.110	0.144
250	12	19.989	27.933	0.062	0.087	3.783	5.237	0.452	0.486	0.127	0.171
190	14	11.391	21.686	0.111	0.152	2.722	4.983	0.155	0.261	0.202	0.277
200	14	27.382	39.251	0.090	0.122	5.134	7.208	0.457	0.510	0.234	0.283
210	14	22.972	19.323	0.100	0.138	4.543	4.408	0.355	0.247	0.161	0.221
220	14	60.816	63.612	0.090	0.132	9.718	10.525	0.966	0.719	0.091	0.145
230	14	90.702	132.600	0.079	0.107	12.998	17.159	1.511	1.509	0.136	0.169
240	14	51.498	67.160	0.073	0.100	8.438	10.603	1.020	0.965	0.092	0.132
250	14	52.219	70.980	0.071	0.098	8.536	11.061	1.065	1.022	0.095	0.138
190	16	17.520	24.548	0.098	0.135	3.623	5.185	0.268	0.310	0.200	0.278
200	16	25.900	24.594	0.108	0.147	5.075	5.373	0.379	0.298	0.188	0.236
210	16	28.896	37.806	0.100	0.137	5.465	7.218	0.448	0.453	0.196	0.251
220	16	54.134	68.870	0.096	0.131	8.936	11.124	0.829	0.773	0.089	0.139
230	16	21.650	30.518	0.086	0.119	4.222	5.935	0.373	0.416	0.119	0.142
240	16	67.361	98.955	0.082	0.113	10.471	14.172	1.150	1.165	0.092	0.124
250	16	36.773	48.408	0.071	0.098	6.430	8.269	0.761	0.740	0.075	0.110

Table A.7: Annualised statistics of out-of-sample tests for sub-interval [2010-12-20, 2016-02-25] of myopic wealth and optimal wealth for the market-neutral constrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 75.089%, volatility of 0.156, expected return of 12.066%, Sharpe ratio of 0.711, and maximum drawdown of 0.186.

Statistics		Profit (%)		Volatility		Expected Return (%)		Sharpe Ratio		Maximum Drawdown	
Windows		$100 \left( \frac{W_T - W_0}{W_0} \right)$		$\frac{1}{\sqrt{\Delta t}} \sigma \left( \frac{\Delta W_t}{W_t} \right)$		$100 \mathbb{E} \left( \frac{\Delta W_t}{W_t} \right)$		$\frac{0.01(\text{Return}-r)}{\text{Volatility}}$		$\max_t \frac{\max_{s \leq t} W_s - W_t}{\max_{s \leq t} W_s}$	
Train	Test	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal	Myopic	Optimal
190	10	189.723	441.757	0.146	0.248	21.716	35.779	1.420	1.404	0.122	0.226
200	10	263.226	664.242	0.160	0.270	26.379	43.121	1.590	1.558	0.091	0.205
210	10	112.127	217.387	0.158	0.260	15.857	25.720	0.943	0.950	0.180	0.293
220	10	128.399	276.686	0.150	0.250	17.261	28.968	1.083	1.117	0.153	0.285
230	10	103.289	179.688	0.187	0.272	15.565	23.665	0.779	0.834	0.224	0.274
240	10	85.314	158.637	0.121	0.193	12.841	20.505	0.983	1.011	0.142	0.192
250	10	102.094	213.065	0.105	0.165	14.387	23.783	1.271	1.382	0.093	0.135
190	12	68.565	169.894	0.253	0.346	12.917	24.384	0.471	0.675	0.415	0.490
200	12	218.773	448.018	0.168	0.250	23.922	36.071	1.362	1.403	0.119	0.163
210	12	100.140	221.550	0.130	0.209	14.386	24.965	1.031	1.148	0.193	0.284
220	12	139.880	307.505	0.136	0.213	18.022	29.684	1.253	1.344	0.118	0.164
230	12	73.414	131.901	0.210	0.309	12.899	21.012	0.566	0.647	0.410	0.514
240	12	70.614	140.306	0.170	0.245	11.902	20.149	0.642	0.781	0.295	0.359
250	12	56.659	73.136	0.140	0.211	9.769	12.929	0.627	0.565	0.194	0.279
190	14	142.118	270.243	0.189	0.307	18.905	29.880	0.946	0.940	0.199	0.269
200	14	279.372	746.858	0.174	0.268	27.408	45.009	1.520	1.643	0.156	0.195
210	14	43.664	80.953	0.140	0.221	8.050	14.041	0.505	0.589	0.350	0.514
220	14	83.293	155.135	0.175	0.249	13.349	21.340	0.707	0.818	0.221	0.273
230	14	86.514	148.774	0.138	0.205	13.158	19.912	0.878	0.924	0.151	0.204
240	14	110.856	277.187	0.152	0.220	15.747	28.341	0.967	1.244	0.161	0.180
250	14	71.984	125.540	0.123	0.196	11.423	17.904	0.844	0.864	0.187	0.320
190	16	132.600	247.545	0.261	0.398	19.283	30.601	0.701	0.743	0.222	0.299
200	16	169.264	348.527	0.197	0.299	21.193	33.532	1.023	1.090	0.187	0.256
210	16	153.224	305.587	0.311	0.410	22.454	34.850	0.690	0.825	0.387	0.468
220	16	79.892	175.559	0.171	0.274	12.928	23.485	0.696	0.820	0.304	0.425
230	16	18.357	38.188	0.206	0.293	5.332	10.419	0.211	0.322	0.439	0.544
240	16	87.000	169.719	0.187	0.285	13.969	23.350	0.693	0.783	0.217	0.321
250	16	52.808	105.563	0.170	0.251	9.889	17.552	0.522	0.661	0.373	0.468

Table A.8: Annualised statistics of out-of-sample tests for sub-interval [2016-02-26, 2021-05-06] of myopic wealth and optimal wealth for the market-neutral constrained portfolios. Risk aversion coefficient is  $\gamma = -70$ , interest rate is  $r = 1\%$ , and factor number is  $m = 6$ . For the S&P 500 ETF, among all the different testing-training window combinations, the statistics for the window with the largest profit are profit of 135.484%, volatility of 0.185, expected return of 18.257%, Sharpe ratio of 0.932, and maximum drawdown of 0.337.